

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS  
APPLICATIONS  
UNIT 3

**SOLUTIONS**

Section Two:  
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (9 marks)

The weight of fish caught by anglers on the first and second day of a -day angling competition was kg and kg respectively.

Let the weight of fish caught on day of the competition be kg.

(a) Assume that the daily weights form an arithmetic sequence.

(i) What weight of fish was caught on the third day of the competition? (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct difference of terms  ü correct weight |

(ii) Deduce a rule that models the weight of fish caught on the day of the competition in the form of (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ express in form from formula sheet: a+(n-1)d  ✓ simplified term rule given in form required |

(iii) What weight of fish was caught on the last day of the competition? (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct weight |

(b) Assume that the daily weights form a geometric sequence.

(i) Determine a recursive rule to model the daily weight of fish caught. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct ratio of terms  ü recursive rule with term of sequence |

(ii) To the nearest kilogram, what weight of fish was caught on the last day of the competition? (2 marks)

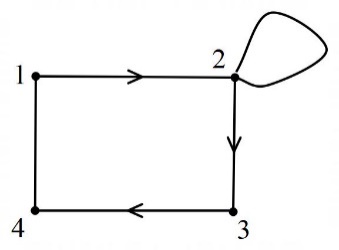
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ with answer unrounded  ü correct weight (rounded to 1dp) |

Question 9 (7 marks)

A harbour has four ferry stations, numbered and . A ferry can only be caught from to , from to , from to , from to and from to . There is also a harbour sightseeing ferry that starts and finishes at station .

(a) Draw directed graph to represent the above information. (2 marks)

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{a421dbba-24d0-4a64-8c32-e85e2b5ae31d}  FXData:  </EFOFEX> OR |
| Specific behaviours |
| ✓ four labelled vertices  ü correct directed graph |



(b) Construct , the adjacency matrix for . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ any matrix filled with zeros and ones  ü correct matrix |

(c) Lani bought an all-day ferry ticket, caught some ferries and ended up at station . Her journey formed a walk of length in graph . Form an appropriate multistage matrix and use it to explain which station(s) she could **not** have started from. (3 marks)

|  |
| --- |
| Solution |
| The zero in the first column indicates that there is no -stage journey starting from station that ends at station , so she could not have started from station . |
| Specific behaviours |
| ✓ calculates  ü explanation using zero in first column  ü states correct station |

Question 10 (5 marks)

A software company is developing two smartphone apps called Cube and Doxa to estimate the percentage sugar content of common foodstuffs from photos taken by the phone’s camera.

The following table shows estimates made using the Cube app and the actual percentage sugar content for eight foodstuffs, where .

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Foodstuff |  |  |  |  |  |  |  |  |
| Estimate |  |  |  |  |  |  |  |  |
| Actual |  |  |  |  |  |  |  |  |

(a) Using the estimate as the explanatory variable, determine the equation of the least-squares line to model the linear relationship between and . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct equation |

When the same eight foodstuffs were used to trial the Doxa app, the equation of the least-squares line to predict the actual sugar content was and .

Cube and Doxa estimated the sugar content of a ninth foodstuff as and respectively.

(b) Use the estimate from each app to predict the actual sugar content of the ninth foodstuff and explain which prediction you have more confidence in. (4 marks)

|  |
| --- |
| Solution |
| Cube: .  Doxa: .  More confident in the Doxa prediction because by comparing the correlation coefficients, the estimates from Doxa have a stronger association with the actual values than the Cube app. |
| Specific behaviours |
| ✓ one correct prediction  ü second correct prediction  ü chooses app with correlation coefficient closest to  ü indicates reasoning based on strength of correlation coefficients |

Question 11 (9 marks)

The balance of savings account after monthly payments of have been made into it can be modelled using the following recurrence relation:

Savings account has an interest rate of per month.

(a) State the balance of the savings account before any interest or monthly payments were made. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct balance |

(b) Determine the balance of the savings account after monthly payments have been made. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ states  ü correct balance, rounded to dp |

(c) Determine , the number of monthly payments that are required for the balance of savings account to first exceed , and state the value of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value of  ü corresponding value of |

The recurrence relation , models the balance of a similar savings account after monthly payments of have been made into it.

(d) State the value of , the monthly payment made into savings account . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

|  |
| --- |
| Solution |
| 1.2% |
| Specific behaviours |
| ✓ correct value |

(e) State the monthly interest rate for savings account . (1 mark)

The difference in the balances of savings accounts and is least after monthly payments have been made into each account.

(f) Determine the value of and the difference between the balances of the accounts at this time. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value of  ü correct difference in balances |

Question 12 (9 marks)

A river runs through a town, separating the northern region from the southern region . The river also splits, forming islands and , and six bridges cross the river at locations shown on the sketch map below.

<EFOFEX>
id:fxd{011b21c8-f7e8-4650-a7e8-63cdff9877ab}

FXData:

</EFOFEX>

(a) Use the information shown in the sketch map to draw a graph in the plane, in which vertices and edges represent the regions and the bridges respectively. (3 marks)

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{f4df159c-d4f6-46d8-b8a9-3adc23f95eed}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ four labelled vertices ü correct graph ü no crossed edges |

(b) A tourist, staying in a hotel on island , wants to take a morning run that starts from their hotel and crosses every bridge in the town once. They don’t mind where their run ends.  
If possible, describe how they can do this. If not possible, explain why not. (2 marks)

|  |
| --- |
| Solution |
| It is possible as graph is semi-Eulerian (starts at and ends at .  Example route: . |
| Specific behaviours |
| ✓ indicates run is possible  ü lists suitable route, ending at or states semi-Eulerian starting at and ending at |

(c) Planning is underway for a seventh bridge in the town. Explain how your answer to  
part (b) would change if the new bridge was a second river crossing between

(i) regions and . (2 marks)

|  |
| --- |
| Solution |
| Still possible - graph now has four even vertices and becomes Eulerian, so will start and end at . |
| Specific behaviours |
| ✓ indicates run ends at start point (W)  ü explains difference referring to network features |

(ii) regions and . (2 marks)

|  |
| --- |
| Solution |
| Run is not possible - graph has more than two odd vertices or no Euler trail or cycle exists. |
| Specific behaviours |
| ✓ indicates run is not possible  ü explains answer using network terminology |

Question 13 (7 marks)

A senior teacher collected data over a -hour period on hours of sleep and hours of mobile phone use from a random sample of students in their school. The coefficient of determination between the variables was calculated to be and the equation of the least-squares line that fitted the data was .

(a) State what percentage of the variation in the hours of sleep of these students can be explained by the variation in their hours of mobile phone use. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct percentage |

(b) Determine the correlation coefficient between and for this data, and hence assess the strength of the association between these variables. (3 marks)

|  |
| --- |
| Solution |
| Therefore, a strong association |
| Specific behaviours |
| ✓ indicates square root of ()  ü uses slope of line to obtain correct coefficient (-ve)  ü assess strength as strong |

(c) Use the least-squares line to predict the hours of sleep for a student who used their mobile phone for hours over a -hour period. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct prediction |

(d) Discuss the validity of the prediction in part (c). (2 marks)

|  |
| --- |
| Solution |
| The strong association between the variables supports its validity.  However, the lack of raw data means it is not possible to determine if the prediction involves extrapolation and so the prediction should be treated with caution. |
| Specific behaviours |
| ✓ indicates strong association supports validity  ü indicates difficulty posed by lack of raw data |

Question 14 (9 marks)

A company specialises in transporting pianos, charging an amount equal to the cheapest path that exists between any pair of cities in the network shown below. Each edge weight represents the cost, in hundreds of dollars, to transport a piano along that edge.

<EFOFEX>
id:fxd{180f94b6-d00d-4522-a923-998ee48aa259}

FXData:

</EFOFEX>

(a) Determine how much the company will charge, and the associated path, to transport a piano between cities

(i) and . (3 marks)

|  |
| --- |
| Solution |
| Example paths:  Cheapest path is and cost is . |
| Specific behaviours |
| ✓ any correct path with cost  ü correct cheapest path  ü correct charge |

(ii) and . (3 marks)

|  |
| --- |
| Solution |
| Example paths:    Cheapest path is and cost is . |
| Specific behaviours |
| ✓ any correct path with cost  ü correct cheapest path  ü correct charge |

(b) Determine the increase in the amount charged to transport a piano between cities and if the company raised every cost shown on the network by . A copy of the original graph has been provided below. (3 marks)

<EFOFEX>
id:fxd{180f94b6-d00d-4522-a923-998ee48aa259}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| Example paths:  Cheapest path now and  cost is ,  an increase of . |
| Specific behaviours |
| ✓ indicates correct method (updates costs or modifies previous paths)  ü indicates new cheapest path or cost  ü states correct increase |

Question 15 (12 marks)

The number of years of experience and the current salary , in thousands of dollars, is shown in the table below for a sample of librarians who belong to a professional association.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (years) |  |  |  |  |  |  |  |  |  |
| () |  |  |  |  |  |  |  |  |  |

(a) Use your calculator to construct a scatterplot of the data with as the explanatory variable and hence describe the nature of the relationship between the variables. (2 marks)

|  |
| --- |
| Solution |
| The scatterplot shows a strong, positive and linear relationship exists between the variables. |
| Specific behaviours |
| ✓ describes nature using at least one of strength, direction or form  ü describes nature using strength, direction and form |

(b) Determine the equation of the least-squares line that can be used to predict from and state the correlation coefficient. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses supplied variables  ü correct coefficients for line (at least 2dp)  ü correct correlation coefficient |

(c) Interpret, in the context of this question,

(i) the slope of the least-squares line. (2 marks)

|  |
| --- |
| Solution |
| For every extra year of experience, a librarian can expect their salary to increase by an average of . |
| Specific behaviours |
| ✓ refers to correct variables  ü correctly states average salary increase according to coefficient in (b) |

(ii) the intercept of the least-squares line. (1 mark)

|  |
| --- |
| Solution |
| The average salary of a librarian with no experience would be . |
| Specific behaviours |
| ✓ correct interpretation |

(d) What percentage of the variation in current salary of these librarians can be explained by the variation in number of years of experience? (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct percentage that rounds to 92% |

(e) Another librarian who belonged to the same professional association has years of experience.

(i) Use the least-squares line to predict the salary of this librarian. (1 mark)

|  |
| --- |
| Solution |
| Salary would be .  NOTE: from CLASSPAD ($116 747) |
| Specific behaviours |
| ✓ correct prediction using equation from (b) or value from CLASSPAD |

(ii) Is the prediction in part (e)(i) reliable? Justify your response. (2 marks)

|  |
| --- |
| Solution |
| No – the prediction involves extrapolation beyond the range of data collected and so cannot be relied upon. |
| Specific behaviours |
| ✓ responds with no  ü justifies response by referring to extrapolation |

Question 16 (9 marks)

The volume of oil wasted, litres, during the trial of a new refining process is given by

(a) Use the recurrence relation to complete the following table. (2 marks)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| (L) |  |  |  |  |  |  |  |

**(16.57) (22.401) (18.3193)**

(b) Add a suitable scale to the vertical axis and then display the volume wasted in each trial on the axes below. (3 marks)

|  |
| --- |
| Solution (a) |
| See table |
| Specific behaviours |
| ✓ at least correct values  ü all correct values (doesn’t have to be rounded to 1dp) |

<EFOFEX>
id:fxd{11844297-e2fc-4f72-9a6f-6f2ed0385ecc}

FXData:

</EFOFEX>

|  |
| --- |
| Solution (b) |
| See graph |
| Specific behaviours |
| ✓ adds suitable scale  ü at least correct points  ü all points correct, no lines |

(c) Will the volume wasted in the twelfth trial be more or less than in the tenth trial? Explain how you can answer this question without calculating or . (2 marks)

|  |
| --- |
| Solution |
| The volume will be more. The graph shows that the volume wasted always increases from any even numbered trial to the next even numbered trial. |
| Specific behaviours |
| ✓ indicates weight will be more  ü explanation using pattern from table or graph |

(d) Describe how the volume wasted changes in the long term. (2 marks)

|  |
| --- |
| Solution |
| The volume wasted gradually converges to a long-term steady state value of L. |
| Specific behaviours |
| ✓ indicates convergence to steady state  ü indicates correct long-term value |

Question 17 (7 marks)

The data in the following table shows , the size of a country’s iron ore stockpile in millions of tonnes and , the price paid by that country to import iron ore in dollars per tonne.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (Mt) |  |  |  |  |  |  |  |  |  |
| ($) |  |  |  |  |  |  |  |  |  |

The data has a correlation coefficient of and the equation of the least-squares line is  
.

(a) Show how to use the point to calculate its residual of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ shows calculation for  ü shows calculation for residual |

(b) Complete the residual plot below. (3 marks)

<EFOFEX>
id:fxd{3e049226-7753-4f11-aef6-206db06fc061}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| Missing residuals:  See graph |
| Specific behaviours |
| ✓ indicates missing residuals  ü correctly plots two residuals  ü correctly plots all residuals |

(c) Based on the residual plot, is it appropriate to fit a linear model to the data? Justify your answer. (2 marks)

|  |
| --- |
| Solution |
| No, it is not appropriate to fit a linear model to the data because a pattern is evident in the residual plot. |
| Specific behaviours |
| ✓ states no  ü justifies with reference to pattern in residual plot |

Question 18 (8 marks)

Fleur suspected a small dam on her farm had a leak and began to monitor the depth of water every hour. The depth of water was initially m and subsequent readings indicated it was decreasing by every hour.

Let be the depth of water in the dam hours after monitoring began.

(a) Write an exponential equation in the form to model the depth of water in the dam.

(2 marks)

|  |
| --- |
| Solution |
| Decay rate: |
| Specific behaviours |
| ✓ indicates correct value for or  ü correct equation |

(b) Determine the depth of water in the dam days after monitoring began. (2 marks)

|  |
| --- |
| Solution |
| days is hours. |
| Specific behaviours |
| ✓ indicates correct value of  ü correct depth, to at least dp |

The leak was caused by a damaged valve. At the instant Fleur fixed the valve, exactly days after monitoring began, heavy rainfall in the region caused the depth of water in the dam to abruptly change from decreasing by to increasing by every hour.

(c) Determine the depth of water in the dam days after monitoring began. (4 marks)

|  |
| --- |
| Solution |
| Depth when  Growth rate:  New model for depth hours after reaching this depth is  Depth when is m. |
| Specific behaviours |
| ✓ indicates depth at end of decreasing phase  ü indicates growth ratio  ü indicates new equation to model increasing depth  ü correct depth (2 marks if correct depth without working) |

Question 19 (7 marks)

Ginevra has created graph to represent a social network within the technology company that she works for. The graph has vertices and at least one vertex of degree .

(a) Give two reasons why the value of cannot be . (2 marks)

|  |
| --- |
| Solution |
| Any vertex degree must be a whole number.  The number of vertices can’t be , but must be at least . |
| Specific behaviours |
| ✓ states degree must be whole number  ü indicates number of vertices and states must be positive |

Three more properties of are that it is a connected planar graph, it has edges and it has faces.

(b) Determine the value of . (3 marks)

|  |
| --- |
| Solution |
| Connected planar graph means that .  Hence |
| Specific behaviours |
| ✓ indicates graph must obey Euler’s formula  ü correctly substitutes into Euler’s formula  ü solves for |

(c) State, with justification, whether is Eulerian. (2 marks)

|  |
| --- |
| Solution |
| No – has a vertex of degree  To be Eulerian all its vertices must be even. |
| Specific behaviours |
| ✓ states no  ü justification for not being Eulerian AND mentions one vertex of degree 7. |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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